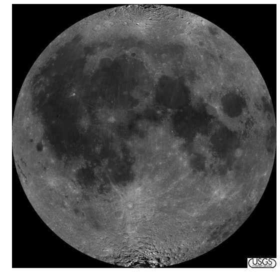
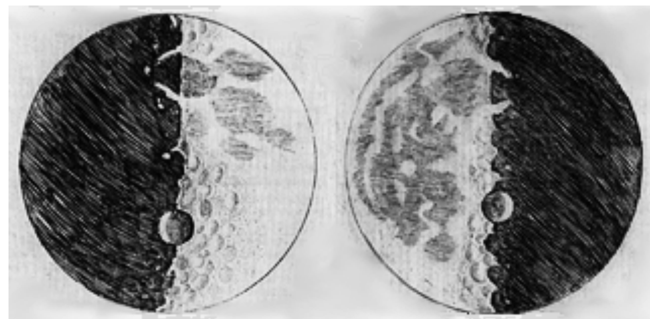
The Height of the Moon's Features



Galileo, turning his telescope to the Moon in 1610, was the first to show that the Moon was not flat, but had projections, mountains and craters. In this exercise we will calculate the height of some of the lunar features. When the Moon is full, the Sun is behind us and we don't see shadows on the moon. But at first or third quarter phase, when the Moon is half-full, the slant sunlight shows features on the terminator (the boundary between the light and dark parts of the moon), which can dramatically show their contours.



Galileo's sketch of first quarter and third quarter moons. The first quarter terminator is sunrise; the third quarter terminator is sunset. (from the Starry Messenger)

Have you ever used a shadow length to estimate the height of something? If you know the actual height of something (or someone) and can measure their shadow's length, then you can calculate the height of something else by measuring its shadow length.

In the picture to the left, assume our girl is 1.1 meters high. What would that be in inches? Remember 1 inch = 2.54 cm. You can always multiply a number by ONE and get the same answer. We can just figure out a clever way to multiply by one to change the units! In this case we can use 1 = (100 cm/ 1m) and also (1 inch / 2.54 cm). So in her case, 1.1 meters = 110 cm. Multiplying now by "1", 110 cm x (1 in / 2.54 cm) = 110 in / 2.54 = 43 in. Notice how the "cm" units cancel each other – yea!

Ok, so if the girl is 1.1 m high, how tall is her shadow here?_______

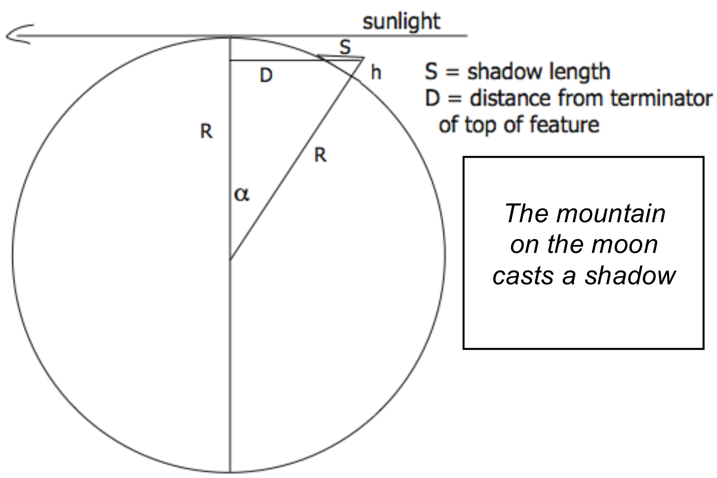
How tall is the flagpole's shadow?___________

Let's use a PROPORTION. The ratio of her height h to her shadow length s is the SAME as the ratio of the flagpole's height H to its shadow length S.
$h/s = H/S$

So, multiplying both sides of the equation by S, we get
$(h * S) / s = H = h * (S/s)$. So we just measure S ________and s _____ (the shadow lengths on this page), divide them, and multiply by the height of the girl (1.1 m). What height do you get for the pole? (you may use a calculator): __________________

Let's now calculate the height of a feature on the Moon!

If a crater rim or mountain is in sunlight, it casts a shadow. We use the length of the shadow to tell us the height of the feature. We use the fact that "similar triangles" (two triangles that have the same angles as each other) have sides that are ratios of each other. We can then solve the problem using proportions!



The ratio of the sides of the big triangle is D/R (where D is the distance from the terminator to the feature, as seen from Earth), but it's also APPROXIMATELY equal to h/S, the height of the feature divided by the shadow length as seen from above. So that makes the calculation especially simple,
H = S * D / R

First, you need to figure out how many cm of your page (use the picture on page 4) corresponds to how many kilometers on the Moon. We do this first by measuring r, the measured RADIUS of the moon (the horizontal segment between the center small circle and the right edge).

How many cm is it on the page? _________.

If we know that the radius of the Moon R is really 1734 km, then we can calculate the scale factor f = 1734 km / r. What do you get for the scale factor? ________(km/cm)

Now, we measure carefully the lengths on the lunar image (page 4). What do you measure

as the distance of a feature from the terminator d_______ and its shadow height s _______ ?

But now we need to use the scale factor f to get the REAL distance D = f * d = ____________

And so we can get the REAL shadow length S = f * s = __________

Only one thing more to do! Calculate the true height of the mountain H = S * D / R.

What do you get?___________

That height is in kilometers, or a thousand meters. Remember that 1 meter is about 3.3 feet,

how many feet tall is that mountain? _____________

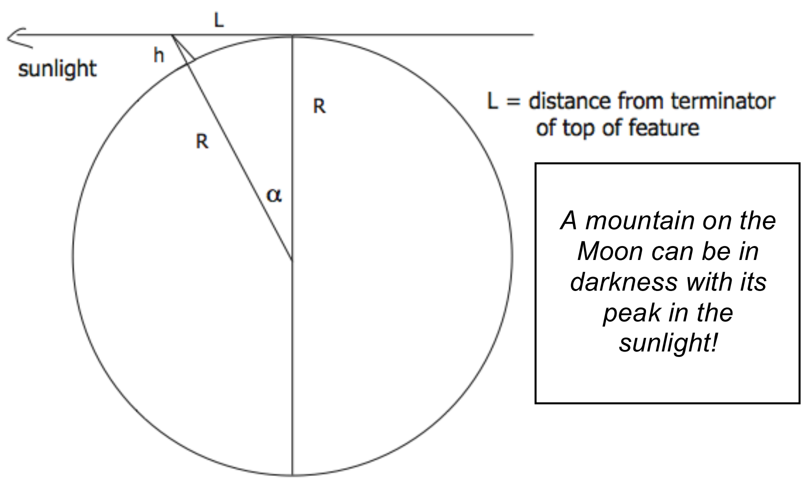
Is that high for a mountain on Earth?

What is the tallest mountain you know on Earth? ______________________

Have you ever been to a mountain that high? Where?

Poking into Sunshine…

When a crater or mountain is just beyond the edge of the moon's shadow (the terminator), the base of the feature may be in darkness but the top of the feature may be in sunlight! That gives an easy way to calculate the minimum height of the feature. (Why is it a minimum?) Note in this diagram, we are looking from the top of the page, so L is the distance we measure from the terminator to the bright spot. This only works if the Moon is near half-full.



Again we can use SIMILAR triangles. The triangle with (R + H) as the hypotenuse is similar to the triangle with L as the hypotenuse. (The base of that triangle is the curved lunar surface, though, so this only works is the angle alpha is very small and the triangle is very skinny).

Again by ratios of similar triangles,

H / L = L / (H + R) ~ L / R (since h is small)

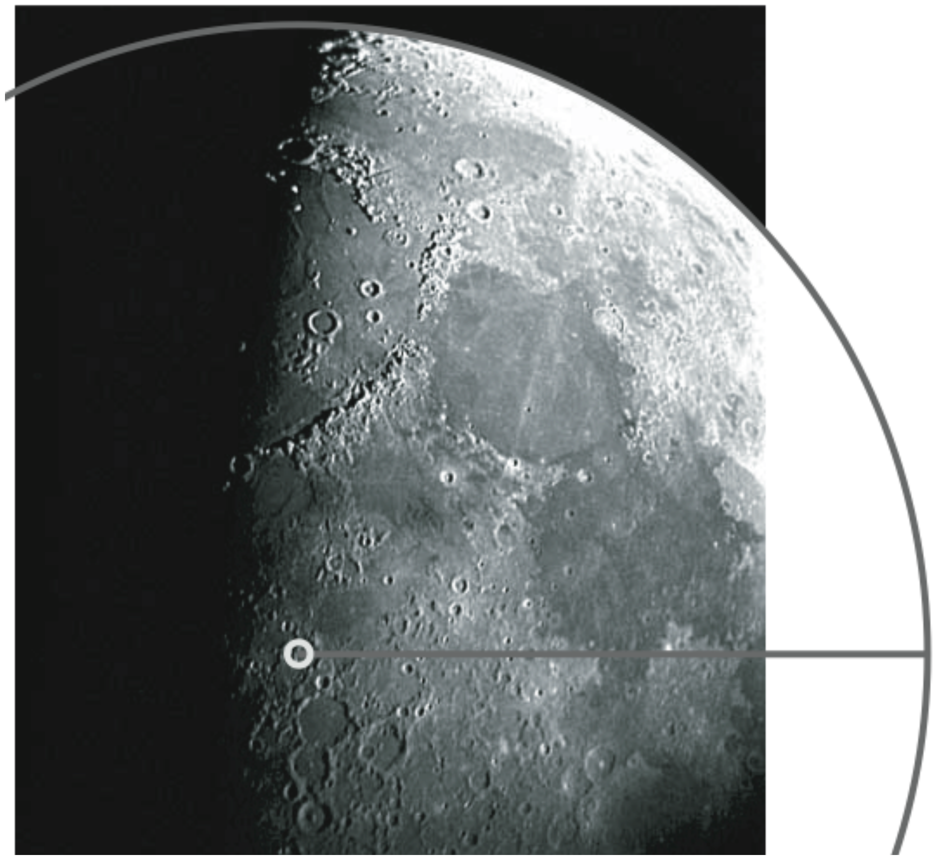
So, measure L' (the distance to the bright spot FROM the terminator): _________ (cm)

Times our scale factor f gives the TRUE distance L in km: ______________ (km)

So we can now calculate H = L * L / R. What do you get? _______________ (km)

And now for a real picture of the moon we can use for these calculations!

Be sure to measure shadow lengths and distances **parallel to the equator line**.
Be sure to measure the length of the lunar radius ON THIS DIAGRAM as well, so you can calculate the scale size to be used to get real lengths.



Lunar image courtesy Loyd Overcash (used by permission)

Time to measure and calculate:

Distance	**Measured length (or calculated)**	**True Length**	**Scale factor (km/cm)**
Lunar radius R (center to edge)		1734 km	
Distance to peak on sunlit side of terminator D			(use the factor calculated above)
Length of shadow S of that peak			
Calculated height of peak on sunny side of terminator h			
Distance L to sunlit peak in dark side of terminator			
Calculated height of sunlit peak h			